

Spin one p -spin glass: the Gardner transition.

T.I. Schelkacheva

Institute for High Pressure Physics, Russian Academy of Sciences, Troitsk
142190, Moscow Region, Russia

E.E. Tareyeva

Institute for High Pressure Physics, Russian Academy of Sciences, Troitsk
142190, Moscow Region, Russia

Abstract. We examine the phase diagram of the p -spin mean field glass model in the spin one case, that is when $S = 0, +1, -1$. For large p the model is solved exactly. The analysis reveals that the phase diagram is in some way similar to that of Ising spins. However, as we show, the quadrupolar regular ordering as well as quadrupolar glass order are present now. The temperature of the Gardner 1RSB – FRBSB transition is obtained explicitly for large p . The case of higher spins is discussed briefly.

1. Introduction

The p -spin spin-glass model of p randomly interacting Ising spins was introduced [1, 2] as a natural generalization of the Sherrington-Kirkpatrick model [3]. In the p -spin model there exists temperature interval where the first step replica symmetry breaking (1RSB) solution is stable. In mean field pure p -spin spherical glasses [4] this interval extends to zero temperature and the transition from replica symmetric (RS) to 1RSB solution is jumpwise in the glass order parameter. It is not so the case when the model contains terms with different values of p [5]. The papers on 1RSB spin glasses make now the basis for so called equilibrium approaches to real glasses (see for reviews [6, 7, 8]).

In discontinuous mean field Ising spin p -spin glass of [1, 2] the mentioned interval of 1RSB stability is finite and increases with increasing p . It is zero for $p = 2$ [3]. The low-temperature boundary of the 1RSB stability region is given by the so called Gardner transition temperature [2]. Recently it was shown [9, 10, 11] that the Gardner transition plays an important role in the equilibrium theory of structural glasses at high pressures near jamming transition. Jamming transition takes place at pressures higher than the Gardner pressure. One can say that at high pressure it is just non-spherical p -spin glasses that are the prototypes of glassy behavior. At very low temperatures in the case of spin glasses or at very high pressure in the case of structural glasses the slow dynamics is determined by the state landscape of FRSB [12].

The detailed work by Rizzo [13] and our own experience in the investigation of arbitrary operator p -operator spin-glass-like models [14, 15, 16, 17, 18] as well as of Potts spin-glass models [19, 20] bring us to the conclusion that Gardner transition is the phenomenon which is common for a large class of models, although explicit analytical result, as far as we know, was obtained in only one case [2]. So, in the present paper we investigate this problem in the case of the spin one p -spin glass.

The p -spin spin glass with $S = 0, \pm 1$ presents the particular case of the problems considered in papers [21, 22] and so a part of our results we obtain now repeat theirs. However, the authors of [21, 22] have paid no attention to the stability of 1RSB solution and the Gardner transition and to the existence of quadrupolar glass in the case of pure p -spin interaction. The investigation of these two problems presents a goal of the present paper.

The paper is organized as follows. In sect.2.1 the model is described and the main equations obtained by replica approach are obtained. In sect.2.2 the case of infinite and that of large p are investigated. Replica symmetric solution as well as the first step replica symmetry breaking solution are considered. The stability of 1RSB solution is examined for large p . The low-temperature instability (Gardner transition) point is obtained explicitly in analytical way. In sect.2.3 for large p the existence of quadrupole orientational glass is demonstrated. In sect.3 the analogous results for higher spin models are derived and discussed.

2. Spin one p -spin glass model

2.1. Main equations

Let us consider the p -spin model with the Hamiltonian

$$H = - \sum_{i_1 \leq i_2 \leq \dots \leq i_p} J_{i_1 \dots i_p} \hat{S}_{i_1} \hat{S}_{i_2} \dots \hat{S}_{i_p}, \quad (1)$$

Here \hat{S} now is the diagonal spin one operator ($S = 0, +1, -1$), N is the number of lattice sites, $i = 1, 2, \dots, N$, and p is the number of interacting particles. $J_{i_1 \dots i_p}$ are independent random variables with Gaussian distribution

$$P(J_{i_1 \dots i_p}) = \frac{\sqrt{N^{p-1}}}{\sqrt{p!}\pi J} \exp \left[-\frac{(J_{i_1 \dots i_p})^2 N^{p-1}}{p!J^2} \right]. \quad (2)$$

Using replica approach we can write the free energy averaged over disorder in the following form that we write here because it is instructive to compare our case with the free energy of the random p -spin model in the case of Ising spins [1, 2]:

$$\begin{aligned} \frac{F}{NkT} = \lim_{n \rightarrow 0} \frac{1}{n} \max \left[-\frac{t^2}{4} \sum_{\alpha} \left(\frac{2+y_{\alpha}}{3} \right)^p + t^2 \sum_{\alpha} \mu_{\alpha} \frac{2+y_{\alpha}}{3} - \right. \\ \left. \frac{t^2}{4} \sum_{\alpha \neq \beta} (q^{\alpha\beta})^p + t^2 \sum_{(\alpha\beta)} q^{\alpha\beta} \lambda_{\alpha\beta} - \ln \text{Tr} \exp \hat{\theta} \right] \end{aligned} \quad (3)$$

with

$$\hat{\theta} = t^2 \sum_{(\alpha\beta)} \lambda_{\alpha\beta} \hat{S}_{\alpha} \hat{S}_{\beta} + t^2 \sum_{\alpha} \mu_{\alpha} \frac{2+\hat{Q}_{\alpha}}{3}.$$

\hat{Q} is the quadrupole-moment operator,

$$(\hat{S}_{\alpha})^2 = \frac{2+\hat{Q}_{\alpha}}{3}$$

in the case $S = 1$, $y_{\alpha} = \langle \hat{Q}_{\alpha} \rangle$ is the regular quadrupole order parameter and $q^{\alpha\beta}$ is the spin glass order parameter; λ, μ are Lagrange multipliers, $t = J/kT$.

Below we will use another form which follows explicitly from our papers where the detailed calculations for the case of interaction of p arbitrary diagonal operators \hat{U} are given (see, e.g.[14]). Let us perform the first stage of the replica symmetry breaking (1RSB) (n replicas are divided into n/m_1 groups with m_1 replicas in each group) and obtain the expression for the free energy. Glass order parameters are denoted by $q^{\alpha\beta} = q_0$ if α and β are from different groups and $q^{\alpha\beta} = q_1$ if α and β belong to the same group. So

$$\begin{aligned} F_{1\text{RSB}} = -NkT \left\{ m_1 t^2 (p-1) \frac{q_0^p}{4} + (1-m_1)(p-1) t^2 \frac{(q_1)^p}{4} - t^2 (p-1) \frac{w_1^p}{4} + \right. \\ \left. \frac{1}{m_1} \int dz^G \ln \int ds^G \left[\text{Tr} \exp \left(\hat{\theta}_{1\text{RSB}} \right) \right]^{m_1} \right\}. \end{aligned} \quad (4)$$

Here

$$\begin{aligned} \hat{\theta}_{1\text{RSB}} = zt \sqrt{\frac{pq_0^{(p-1)}}{2}} \hat{S} + st \sqrt{\frac{p[(q_1)^{(p-1)} - q_0^{(p-1)}]}{2}} \hat{S} + \\ t^2 \frac{p[w_1^{(p-1)} - (q_1)^{(p-1)}]}{4} \hat{S}^2, \end{aligned} \quad (5)$$

$$\int dz^G = \int_{-\infty}^{\infty} \frac{dz}{\sqrt{2\pi}} \exp \left(-\frac{z^2}{2} \right). \quad (6)$$

The extremum conditions for F_{1RSB} yield equations for the order parameters. We get glass order parameters q_0 and q_1 , the auxiliary order parameter w_1 , the regular order parameter x_1 and the parameter m_1 . Auxiliary order parameter w_1 arises from the fact that \hat{S} in Eq. (1) are not Ising spins.

$$q_0 = \int dz^G \left\{ \frac{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{(m_1-1)} [\text{Tr } \hat{S} \text{ exp } \hat{\theta}_{\text{1RSB}}]}{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{m_1}} \right\}^2, \quad (7)$$

$$q_1 = \int dz^G \frac{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{(m_1-2)} [\text{Tr } \hat{S} \text{ exp } \hat{\theta}_{\text{1RSB}}]^2}{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{m_1}},$$

$$w_1 = \int dz^G \frac{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{(m_1-1)} [\text{Tr } \hat{S}^2 \text{ exp } \hat{\theta}_{\text{1RSB}}]}{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{m_1}}, \quad (8)$$

$$x_1 = \int dz^G \frac{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{(m_1-1)} [\text{Tr } \hat{S} \text{ exp } \hat{\theta}_{\text{1RSB}}]}{\int ds^G [\text{Tr exp } \hat{\theta}_{\text{1RSB}}]^{m_1}}. \quad (9)$$

Similarly, we obtain the equation for the order parameter m_1 .

The corresponding expressions for the RS approximation can be easily obtained from the preceding formulas (4)-(9) when $q_0 = q_1$. We have

$$q_{\text{RS}} = \int dz^G \left\{ \frac{\text{Tr} [\hat{S} \text{ exp } (\hat{\theta}_{\text{RS}})]}{\text{Tr} [\text{exp } (\hat{\theta}_{\text{RS}})]} \right\}^2, \quad (10)$$

$$w_{\text{RS}} = \int dz^G \frac{\text{Tr} [\hat{S}^2 \text{ exp } (\hat{\theta}_{\text{RS}})]}{\text{Tr} [\text{exp } (\hat{\theta}_{\text{RS}})]}, \quad (11)$$

$$x_{\text{RS}} = \int dz^G \frac{\text{Tr} [\hat{S} \text{ exp } (\hat{\theta}_{\text{RS}})]}{\text{Tr} [\text{exp } (\hat{\theta}_{\text{RS}})]}, \quad (12)$$

Here

$$\hat{\theta}_{\text{RS}} = zt \sqrt{\frac{p q_{\text{RS}}^{(p-1)}}{2}} \hat{S} + t^2 \frac{p[w_{\text{RS}}^{(p-1)} - q_{\text{RS}}^{(p-1)}]}{4} \hat{S}^2. \quad (13)$$

2.2. Large p solutions. Gardner transition temperature.

In the case of $p \rightarrow \infty$ the problem can be solved exactly [21, 22]. Consideration of such a limiting case makes it possible to describe many properties of the model for finite values of p .

It is easy to see that order parameters come in $\hat{\theta}$ and F in the form of a power function q^p and w^p . Herewith $0 \leq q \leq 1$ and $0 \leq w \leq 1$.

Let us consider first the replica symmetric case in the limit $p \rightarrow \infty$. Let's pretend that $0 \leq q < 1$ and $0 \leq w < 1$. Then $q^p = w^p = 0$ and we get directly $q_{\text{RS}} = 0$,

$x_{\text{RS}} = \langle \hat{S} \rangle = 0$ and $w_{\text{RS}} = 2/3$ from Eq. (10) - Eq. (13). The ordering of spins is absent.

Quadrupole operator $\hat{Q} = 3\hat{S}^2 - 2 = (-2, 1, 1)$. So a regular quadrupolar ordering is also absent because average value $\langle \hat{Q} \rangle = 3w_{\text{RS}} - 2 = 0$. We have got disordered paraphase with the free energy $F/(NkT) = -\ln 3$.

There is another solution in the replica symmetric consideration. When $0 \leq q < 1$ and $w = 1$ we have $q^p = 0, w^p = 1$. It turns out from Eq. (10) - Eq. (13) that $q_{\text{RS}} = 0, x_{\text{RS}} = 0$ and $w_{\text{RS}} = 1$. Then average values $\langle \hat{Q} \rangle = 3w_{\text{RS}} - 2 = 1$ and $\langle \hat{S} \rangle = 0$. The phase is not ordered in spins, but there is a quadrupole ordering.

It is important to note that the contribution to ordering is given only by the states $S = +1, -1$. The free energy is:

$$F_{\text{RS}}/(NkT) = -J^2/(2kT)^2 - \ln 2. \quad (14)$$

It is identical to the result for the case of Ising spin $\hat{S} = (+1, -1)$ [1, 2].

All these RS states as well as the transitions between them are described in details in [21, 22].

Let us consider now the 1RSB case in the limit $p \rightarrow \infty$. Let us emphasize that now it is the paraphase with the nonzero quadrupolar ordering that bifurcates. In accordance with the Parisi approach [23] we have $q_0 < q_1$. Hence we immediately obtain the order parameters from Eq. (7) - Eq. (9): $q_0 = 0, q_1 = 1, x_1 = \langle \hat{S} \rangle = 0$ and $w_1 = 1$. So we have got a glass phase with a nonzero spin glass order parameter $q_1 = 1$. The quadrupole ordering is preserved $\langle \hat{Q} \rangle = 3w_1 - 2 = 1$. Still contribution is only from $S = +1, -1$. A value of $S = 0$ does not make a contribution.

The expression for the free energy is given by (Eq. (4)) and has the form:

$$F_{\text{1RSB}}/(NkT) = -m_1 J^2/(2kT)^2 - (1/m_1) \ln 2. \quad (15)$$

It coincides with that which was obtained for the Ising spins [1, 2]. The expression for m_1 can be obtained as the extremum condition for $F_{\text{1RSB}}/(NkT)$:

$$m_1^2 J^2/(2kT)^2 = \ln 2. \quad (16)$$

When $m_1 = 1$ free energies Eq. (14) and Eq. (15) become equal. From Eq. (16) we have $kT_c/J = 1/(2\sqrt{\ln 2})$. Since $m_1 J/(2kT)$ is independent of temperature. F_{1RSB} is independent of temperature too below T_c . This is exactly the same results as for the problem with Ising spins [1, 2]. This 1RSB solution was obtained in [22]. The problem of its stability was not considered in that paper. Let us investigate it now.

We can break the replica symmetry in our model Eq. (1) once more and obtain the corresponding expressions for the free energy and the order parameters. The bifurcation condition $\lambda_{(\text{1RSB})\text{repl}} = 0$ determining the temperature $T = T_G$ (the Gardner temperature) of instability follows from the condition that a nontrivial small solution for the 2RSB glass order parameter appears (see [15]). We have:

$$\lambda_{(\text{1RSB})\text{repl}} = 1 - t^2 \frac{p(p-1)(q_1)^{(p-2)}}{2} \times \frac{\int ds^G \left[\text{Tr} \exp(\hat{\theta}_{\text{1RSB}}) \right]^{m_1} \left\{ \frac{\text{Tr}[\hat{S}^2 \exp(\hat{\theta}_{\text{1RSB}})]}{\text{Tr}[\exp(\hat{\theta}_{\text{1RSB}})]} - \left[\frac{\text{Tr}[\hat{S} \exp(\hat{\theta}_{\text{1RSB}})]}{\text{Tr}[\exp(\hat{\theta}_{\text{1RSB}})]} \right]^2 \right\}^2}{\int ds^G \left[\text{Tr} \exp(\hat{\theta}_{\text{1RSB}}) \right]^{m_1}}. \quad (17)$$

Eq. (17) depends only on 1RSB-solution. It has been shown that 1RSB solution is stable when $\lambda_{(\text{1RSB})\text{repl}} > 0$ [2, 14].

When $p \rightarrow \infty$ the 1RSB glass solution is stable, because we have $(2kT)^2 \lambda_{(1RSB)_{\text{repl}}} > 0$ from Eq. (17) at all temperatures $T > 0$. At large but finite values of p the condition $\lambda_{(1RSB)_{\text{repl}}} > 0$ is violated at low temperature $T_G \neq 0$.

Let us calculate the Gardner transition temperature T_G explicitly from the requirement $\lambda_{(1RSB)_{\text{repl}}} = 0$. In our case $\hat{\theta}_{1RSB} = st\sqrt{\frac{p}{2}}$ and

$$\Psi(s) \equiv \text{Tr} \exp \hat{\theta}_{1RSB} = 1 + 2 \cosh st \sqrt{\frac{p}{2}}. \quad (18)$$

We can rewrite the equation for λ in the form:

$$\lambda = 1 - \frac{t^2}{2} p(p-1) \frac{\int ds^G \Psi(s)^{m-4} (3 + \Psi(s))^2}{\int ds^G \Psi(s)^m} \quad (19)$$

Calculating the transition point we keep in mind that at large p the values of T and m are small. So we obtain after deriving the asymptotics of integrals and summing the series the equation for the limit of stability of 1RSB phase:

$$1 = \frac{p^{3/2} t}{4\sqrt{\pi} 2^p} \left(\frac{5}{6} - \frac{2\pi}{9\sqrt{3}} \right), \quad (20)$$

so, that

$$kT_G/J = \frac{0.1076}{\sqrt{\pi}} \frac{p^{3/2}}{2^p}. \quad (21)$$

Let us note that the p -dependence is of the same form as in the case of Ising spins.

2.3. Large p solution. The existence of the quadrupole glass.

The main difference of Ising and spin one cases is the presence of a quadrupole ordering in the latter one. We can define the quadrupole glass (orientational) order parameter by Eq. (7) and Eq. (8) replacing \hat{S} to \hat{Q} and keeping in mind the zero limit of interaction of quadrupoles. The function $\hat{\theta}_{1RSB}$ is not changed at the shutdown of quadrupole-quadrupole interaction.

$$q_0^Q = \int dz^G \left\{ \frac{\int ds^G [\text{Tr} \exp \hat{\theta}_{1RSB}]^{(m_1-1)} [\text{Tr} \hat{Q} \exp \hat{\theta}_{1RSB}]}{\int ds^G [\text{Tr} \exp \hat{\theta}_{1RSB}]^{m_1}} \right\}^2, \quad (22)$$

$$q_1^Q = \int dz^G \frac{\int ds^G [\text{Tr} \exp \hat{\theta}_{1RSB}]^{(m_1-2)} [\text{Tr} \hat{Q} \exp \hat{\theta}_{1RSB}]^2}{\int ds^G [\text{Tr} \exp \hat{\theta}_{1RSB}]^{m_1}}. \quad (23)$$

In the limit of infinite p we obtain quadrupole glass order parameters $q_0^Q = 1$ and $q_1^Q = 1$. But we can not say that, along with the spin glass we obtain a quadrupole glass, because $(\langle \hat{Q} \rangle_{1RSB})^2 = q_0^Q = q_1^Q = 1$.

To clarify the question of the presence or absence of quadrupole glass let us consider now the case of large but finite values of p . It is suitable to write the equations for $\langle \hat{Q} \rangle_{1RSB}$ and q_1^Q as follows:

$$\langle \hat{Q} \rangle_{1RSB} = 1 - 3 \frac{\int ds^G \Psi(s)^{m-1}}{\int ds^G \Psi(s)^m}$$

$$q_1^Q = 1 - 6 \frac{\int ds^G \Psi(s)^{m-1}}{\int ds^G \Psi(s)^m} + 9 \frac{\int ds^G \Psi(s)^{m-2}}{\int ds^G \Psi(s)^m}$$

and we have

$$q_1^Q - \langle \hat{Q} \rangle_{RSB}^2 = 9 \left[\frac{\int ds^G \Psi(s)^{m-2}}{\int ds^G \Psi(s)^m} - \left(\frac{\int ds^G \Psi(s)^{m-1}}{\int ds^G \Psi(s)^m} \right)^2 \right]. \quad (24)$$

Now we can proceed as when obtaining the Gardner temperature. At large p the integrals $\int ds^G \Psi(s)^\eta$ with $\eta > 0$ are approximately equal to $2 * 2^p$ while those for $\eta < 0$ are proportional to $1/\sqrt{\pi p}$. So, the considered difference is

$$q_1^Q - \langle \hat{Q} \rangle_{RSB}^2 = 9 \left[\frac{\Sigma_1}{\sqrt{\pi p} 2^p} - \left(\frac{\Sigma_2}{\sqrt{\pi p} 2^p} \right)^2 \right] > 0 \quad (25)$$

with Σ_i standing for converging sums that can be easily evaluated.

This means that for large but finite p the quadrupolar orientational glass is present along with the spin glass in spin one system.

Such a phenomenon was first encountered in generalized SK [3] model for spin one case investigated in RS approximation in the paper Ref. [24]. After performing high-temperature series expansion of the RS equations Eq. (10) - Eq. (13), one easily makes sure that average value of quadrupole $\langle \hat{Q} \rangle_{RS} = 3w_{RS} - 2$ is different from zero at arbitrarily high temperatures, so random distribution of spins \hat{S} produces a non-zero average value of the quadrupole moment, which gradually increases with decreasing temperature to a critical temperature T_c . At temperatures below T_c , the system continues to have quadrupole ordering. At the point T_c appears spin glass and quadrupole glass, too, i. e., the order parameter defined in the replica symmetric consideration by the relation

$$q_{RS}^Q = \int dz^G \left\{ \frac{\text{Tr} [\hat{Q} \exp(\hat{\theta}_{RS})]}{\text{Tr} [\exp(\hat{\theta}_{RS})]} \right\}^2, \quad (26)$$

ceases to be equal to $[\langle \hat{Q} \rangle_{RS}]^2$. We define the parameters of the quadrupole glass in the limit off quadrupole interaction.

Low-temperature asymptotic behavior of the order parameters may be obtained analytically:

$$\langle \hat{Q} \rangle_{RS} = 1 + O(e^{-t}), q_{RS}^S = 1 - \frac{2}{3\sqrt{2\pi}} t^{-1} + O(e^{-t}). \quad (27)$$

The most interesting of these results is the fact that $\langle \hat{Q} \rangle_{RS} = 1$ and $q_{RS}^S = 1$ at $T = 0$. This means that at zero temperature, all spins take only the values +1 and -1. The state $S = 0$ is absent. It may be used to determine the number of metastable states $\langle N_S \rangle = e^{0.19923N}$ at zero temperature. It is known result for the SK model [25]. So that our spin one glass is quite similar to the Ising spin glass at $T = 0$.

3. Higher spins p -spin glass

Consider now the case of Hamiltonian Eq. (1) with larger spin values $j = 2, 3, \dots$. We will use normalized operators $\hat{S} = \hat{j}_z/j$ that is much more convenient for calculations and does not change the symmetry of the problem. So when $j = 1$ we have

$(1, 0, -1)$ as before. For $j = 2$ we use $\frac{1}{2}\hat{j}_z = \frac{1}{2}(2, 1, 0, -1, -2)$. For $j = 3$ we use $\frac{1}{3}\hat{j}_z = \frac{1}{3}(3, 2, 1, 0, -1, -2, -3)$ and so on.

As is known, the quadrupole moment is $\hat{Q} \sim [3\hat{j}_z^2 - j(j+1)]$ in the space $j = \text{const.}$ We will use normalized expression for quadrupole moment $[3\hat{j}_z^2 - j(j+1)]/j^2$ for uniformity of computing that does not change the symmetry of the problem.

We operate on completely similar to the previous case of spin one. First of all, we get completely disordered paraphase. We have not glass $q_{RS}^S = 0$. There is no ordering of the spins. We obtain $0 < w_{RS} < 1$: $w_{RS} = 2/3$ for $j = 1$, $w_{RS} = 1/2$ for $j = 2$, $w_{RS} = 4/9$ for $j = 3$, $w_{RS} = 5/12$ for $j = 4$. For average value $\langle \hat{Q}_j \rangle = 3 \langle (\hat{j}_z/j)^2 \rangle - (j+1)/j = 3w_{RS} - (j+1)/j$ we get $\langle \hat{Q}_{j=1} \rangle = \langle \hat{Q}_{j=2} \rangle = \langle \hat{Q}_{j=3} \rangle = \dots = 0$. Hence, we have no quadrupole ordering. Free energy is $F/(NkT) = -\ln(2j+1)$, so $F/(NkT) = -\ln(3)$ for $j = 1$, $F/(NkT) = -\ln(5)$ for $j = 2$, and so on. We obtain from $q_{RS} < 1$ and Eq. (13) that $\hat{\theta}_{RS} = t^2 \frac{w_{RS}^{(p-1)}}{4} \hat{S}^2$ where $\hat{S} = \hat{j}_z/j$. So we get from $w_{RS} < 1$ and Eq. (11) that completely disordered phase occurs at a purely formal mathematical case $p \rightarrow \infty$. For arbitrarily large but finite values of p such a phase takes place only at $T \rightarrow \infty$ (or $t \rightarrow 0$).

The results presented below present a strict generalization of that for spin one case. The formulas Eq. (14) - Eq. (16) hold now, too. This is due to the fact that in calculating the integrals of the type $\int ds^G [\text{Tr exp}(s\sqrt{\frac{T}{2}}t\hat{S})]^m$ only the terms with the largest absolute values of the normalized operator $\hat{S} = \hat{j}_z/j$ do contribute. We obtain two low-temperature phases: disordered phase of spin values ($q_{RS} = 0$, $x_{RS} = \langle \hat{j}_z/j \rangle = 0$, $w_{RS} = 1$) and 1RSB spin glass phase ($q_0^s = 0$, $q_1^s = 1$, $x_1^s = \langle \hat{j}_z/j \rangle_{1RSB} = 0$ and $w_1 = 1$). The first of these phases (no glass) for $j \geq 2$ is formally less energetically favorable than paraphase when the temperature decreases from arbitrarily high temperature to a temperature of occurrence of 1RSB spin glass.

Quadrupole ordering occurs in two low-temperature phases: average values are $\langle \hat{Q}_{j=1} \rangle = 1$, $\langle \hat{Q}_{j=2} \rangle = 3/2$, $\langle \hat{Q}_{j=3} \rangle = 5/2$. In 1RSB spin glass phase we obtain from Eq. (22) - Eq. (23) quadrupole glass order parameters $q_0^Q = q_1^Q = [\langle \hat{Q}_{j=1} \rangle]^2 = 1$ for $j = 1$, $q_0^Q = q_1^Q = [\langle \hat{Q}_{j=2} \rangle]^2 = [3/2]^2$ for $j = 2, \dots$. This is a consequence of the fact that only the maximum values of the operator \hat{Q} significantly contribute to $\text{Tr } \hat{Q} \exp(\hat{\theta}_{1RSB})$ under the integral when $p \rightarrow \infty$. So there is no quadrupole glass along with the spin glass in the limit $p \rightarrow \infty$ limit.

4. Conclusions

The phase diagram of the p -spin mean field glass model in the spin one case is examined in details for large values of p . Some new facts, as compared with [21, 22], are established. It is shown that 1RSB phase is unstable at low temperatures and the low-temperature boundary of stability, the Gardner temperature, is calculated explicitly in analytical way. An interesting feature of spin glass phase, as we show, is the existence of quadrupole orientational glass along with the spin glass in addition to known regular quadrupole ordering. The case of higher spins is discussed, too.

It is interesting that the contribution to the values of order parameters comes from all the values of \hat{S} only at high temperatures. At low temperatures, the main contribution is made by the maximum values of the spin operators. It is possible that for other operators and models of cluster glass [26, 27] some components or some

of the spherical harmonics effectively fall out. Perhaps the successful description of the various complex glass systems through a simple Ising spins can be explained by this fact. Note that a complication of interaction (compared to Ising spins) leads to induced hidden regular order and hidden glass for the operators that do not enter the Hamiltonian.

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